TEACHER GUIDE

SURFACE AREA (USING FORMULAS)
GRDES 6–8

COMMON MISCONCEPTIONS

• Students leave variables in the final expression.
If students do not understand the purpose of using variables to generalize an expression, they may not substitute values for the variables in a formula, and may leave them in the final answer. For example, to calculate \( A = l \times w \) where \( l = 4 \) and \( w = 3 \), students may write something like \( A = 4l \times 3w \). Emphasize that \( l \) and \( w \) stand for the values of the length and width. Have students work through several examples and ensure that they can use substitution before moving on.

• Students confuse formulas for the circumference and area of a circle.
It is common for students to confuse the formulas for the circumference and area of a circle. This is particularly tricky in surface area formulas that include both, such as for the surface area of a cone or a cylinder. Show students that we can differentiate the formulas by noting that the area formula uses \( r^2 \) while the circumference formula only uses \( r \). When we find the area of one face of a shape, we need to multiply two dimensions together. \( r^2 \) means \( r \times r \), which is two dimensions. \( r \) is only one dimension. Remind students that even though \( \pi \) looks like a variable, it stands for a specific number.

• Students mislabel or do not label drawings.
Formulas can only be used if drawings are labeled appropriately. This is especially important where students need to calculate the area of many different shapes (such as 3 rectangles in prisms) or where the base of one shape is the same as the length or height of another. Ensure that students consistently match the labels on their drawings to the variables in their formulas. They should not use one variable to label two different dimensions. This is particularly tricky with triangular prisms, where the “base” of the triangles is also one of the rectangle dimensions. Remind students that each letter can only stand for one side.

SURFACE AREA OF RECTANGULAR PRISMS

Students first learn the simple formula for a cube’s surface area before learning the more general formula for all rectangular prisms. Even though the focus is on formulas here, the aim is for students to generate the formulas on their own through scaffolding provided by the lesson and manipulatives. Ensure that students can see and touch foldable cubes and rectangular prisms that should be, if possible, color coded to indicate which faces have the same area. For a cube, all faces should have the same color, and because there are 6 faces, the surface area is \( 6a^2 \). For rectangular prisms, we have 3 pairs of matching faces, and we have up to three different side dimensions for length, width, and height. Give students examples where all three dimensions are different, two dimensions have the same length, and all three dimensions have the same length (a cube).
Check in to ensure they understand that a cube is a type of rectangular prism. Students may notice that $2(l \times w) + 2(w \times h) + 2(l \times h)$ is the same as $2[(l \times w) + (w \times h) + (l \times h)]$ and they should be invited to share this discovery with the class.

SURFACE AREA OF CYLINDERS AND TRIANGULAR PRISMS

To teach surface area of cylinders, ensure that you have foldable 3D models or tubes with lids that can be unrolled. To help students connect that the circumference of the circle at either end is also one of the rectangle dimensions, you can highlight that edge or mark it with a color that stands out. Ensure that students understand why we need to calculate both the circumference and area of the circle as part of the process. Finding the surface area of triangular prisms is not much trickier than for cylinders or rectangular prisms, but it is important that sides and edges are carefully labeled. To find the area of a triangle, we need a “height,” which is not to be confused with the actual height of a 3D triangular prism. To calculate the areas of the 3 rectangles that connect the two triangles, we have up to three different “widths.” For consistency, give the variables distinct names that can be used for all calculations leading up to the total surface area, such $a$, $b$, $c$ as the sides of the triangle, and prism height $d$. Then our formula for surface area looks something like: $(b \times h) + (a \times d) + (b \times d) + (c \times d)$, where students must choose which triangle side is the base, $b$. Only use right triangles, or provide triangle height, since students do not yet know the Pythagorean Theorem. Also ensure that students understand that the $b \times h$ portion of the calculation comes from $2 \times \frac{b \times h}{2}$.

SURFACE AREA OF CONES

Finding the surface area of a cone is more complex, and students may not have seen it derived before. Once again, show students how to derive the area formula using models that can be unrolled. Once a cone is opened, it looks like part of a circle. Before you slice up the partial circle, ensure that students understand that the length of the arc measures $2\pi r$ and that the $r$ refers to the radius of the flat circle at the end of the cone. This can be a challenging connection to make. When you have sliced up the partial circle, highlight the bottom edge using a separate color.

This way, when you assemble your rectangle, students can see that half of the arc went to one length of the rectangle, and the other half went to the opposite length. This is the most challenging concept of the lesson, and students should be given plenty of practice with it. Stress the difference between slant height and height.

TEACHER TIPS

Correct language and careful labeling are essential for success with this lesson. The learning goal of this lesson is not for students to memorize formulas, but to learn how to generalize problems and derive formulas using creative and critical thinking. They should therefore be encouraged to draw and keep 3D models at hand while they are building and using these new formulas, even during assessments. It is a more important indicator of mastery if students can repeatedly show how a formula is created, than if they can simply memorize it and use it. To ensure that you are assessing students more on their process than correct answers, ask students to clearly label the shape before they calculate its surface area, and encourage them to show all their steps, gradually simplifying their expression with each line of math.

In addition, do your best to ensure that you are using clear language. “Side” refers to one dimension of a two-dimensional shape, while “face” indicates a two-dimensional flat part of a three-dimensional object.