COMMON MISCONCEPTIONS

• Side lengths are related by addition or subtraction.
  Students who look for patterns may do so through additive thinking. When students see a 3-4-5 right triangle, they may think that any three consecutive whole-number lengths, such as 5-6-7, make a right triangle. When students see a 6-8-10 right triangle, they may try to conclude that any three consecutive even-number lengths, such as 16-18-20, make a right triangle. Applaud students who search for patterns and remind them that the pattern they need is the Pythagorean Theorem, where the sum of the squares of the shorter two sides is equal to the square of the longest side, or symbolically: $a^2 + b^2 = c^2$. Emphasize this idea by drawing a 3-4-5 right triangle and the corresponding 3 × 3, 4 × 4, and 5 × 5 squares along the sides of the triangle. This clearly shows the pattern $3^2 + 4^2 = 5^2$, because $9 + 16 = 25$.

• Each letter in the Pythagorean Theorem can represent any side of a triangle.
  Although the legs can be interchangeably assigned to $a$ and $b$, the hypotenuse must always be $c$. This misconception happens when the student does not understand that the Pythagorean Theorem is essentially a tool to relate the lengths of the legs to the length of the hypotenuse. When they encounter this error, ask students to determine whether the equation $30^2 + 40^2 = 50^2$ is true or false. They should observe that $900 + 1,600 = 2,500$, so the equation is true. Then, ask students to determine whether the equation $50^2 + 30^2 = 40^2$ is true or false. They should observe that $2,500 + 900 > 1,600$, so the equation is false. This demonstrates the importance of assigning the longest length, the hypotenuse, to the variable $c$. This test also represents a way to verify whether a student’s answer for an unknown side length is reasonable.

• Students rearrange the formula incorrectly when solving for the length of a leg.
  To solve for the length of a leg of a right triangle, students must rearrange the formula after they have substituted in the known numbers. For example, solve for $a$ in $a^2 + 8^2 = 10^2$ or $a^2 + 64 = 100$. When solving, students may forget to apply inverse operations and instead add the value of the square of the given leg to the square of the hypotenuse. To help students avoid this error, remind them that they should always check how realistic their answer is. If they are solving for the length of a leg and get a result that is greater than the length of the hypotenuse, they know that they must have made an error in their process.

TRIANGLE BASICS

In earlier grades, students developed skills that are prerequisite to studying the Pythagorean Theorem.
These skills include measuring lengths; identifying a right angle, the base, and the height; and differentiating between acute, right, and obtuse triangles.

These foundational skills prepare students to calculate, rather than measure, the missing side length in a right triangle when two lengths are known, and to recognize that the longest side of a right triangle is opposite its 90° angle. The base and height of the triangle form this right angle. The terms base and height are essentially interchangeable, because rotating a triangle preserves its side lengths. In this lesson, those sides are generally referred to as the triangle’s legs, said to have length $a$ and length $b$. The longest side, or hypotenuse, is said to have length $c$.

Notice that it is customary for length $a$ to be opposite point $A$, length $b$ to be opposite point $B$, and length $c$ to be opposite point $C$. When students encounter a different variable, such as $x$, to represent an unknown length, they need to decide whether that variable refers to a leg or to the hypotenuse.

**INVERSE OPERATIONS**

Students also learned how to solve two-step equations with inverse operations in earlier grades. The example below is in the form $ax + b = c$, where $a$, $b$, and $c$ are constants.

\[
\begin{align*}
5x + 35 &= 50 \\
5x &= 15 \\
x &= 3
\end{align*}
\]

The two operations in the original equation are multiplication and addition. To solve this equation, we use the inverse operations, division and subtraction, in the reverse order. First, subtract 35 from both sides to isolate the term $5x$. Then, divide both sides by 5 to get $x = 3$. To verify that this solution is correct, substitute 3 for $x$ and simplify the left-hand side of the original equation. The expression $5(3) + 35$ is equivalent to $15 + 35$, which simplifies to 50. So the answer is correct. In this lesson, squaring is one of the operations, as in the example below.

\[
\begin{align*}
5^2 + b^2 &= 13^2 \\
25 + b^2 &= 169 \\
b^2 &= 144 \\
b &= 12
\end{align*}
\]

Notice that, once we have calculated the perfect squares, we subtract 25 from each side to isolate the term $b^2$. The next inverse operation is a square root, changing $b^2$ into $b$, and $\sqrt{144}=12$. 
Understanding the Pythagorean Theorem relies on geometry. The graphic demonstrates why, given a 3-4-5 right triangle, it is true that $3^2 + 4^2 = 5^2$. Notice that 9 unit squares plus 16 unit squares equals 25 unit squares.

In high school, students learn that sets of whole numbers that satisfy the Pythagorean Theorem are called “Pythagorean Triples.” When students use the converse of the Pythagorean Theorem to determine whether a set of three numbers could make a right triangle, they are finding examples and non-examples of Pythagorean Triples.

The examples below show that 7-24-25 is a Pythagorean Triple, while 8-10-12 is not.

$$7^2 + 24^2 = 25^2$$
$$49 + 576 = 625$$
$$625 = 625$$

$$8^2 + 10^2 = 12^2$$
$$64 + 100 = 144$$
$$164 \neq 144$$