DEVELOPMENT OF UNDERSTANDING PLACE VALUE

In Kindergarten and first grade, students begin to think of groups of ten objects as a unit and begin to understand how place value is reflected in two-digit numbers. In grade 2, students expand this understanding to three-digit numbers. In grade 3 and beyond, students work with numbers up to 6 digits and extend their knowledge of the place-value system to six places. They start to see the regularity with which the system works.

COMMON MISCONCEPTIONS

- **Students count the hundreds, tens, and ones as separate numbers.**
  
  When students count a collection of base-ten blocks made up of hundreds, tens, and ones, they count the hundreds, tens, and ones as separate numbers. For example, they may count 423 as 100, 200, 300, 400, 10, 20, 1, 2, 3. When students count like this, they do not see the parts of the number as making up one number. Tell students that when they count, the number they say should tell how many they have counted so far. When they say “10” after “400” they are no longer keeping track of the cardinality of the counted set. If students count this way, pause them as they are counting and ask “how many have you counted in all?”

- **Students see 2- and 3-digit numbers independent of place value.**
  
  When students are asked about the value of the digits in a number, such as 759, they say that the values are 7, 5, and 9. They do not make any connection between the value of the digit and its place in the number. These students need more experience modeling numbers with base-ten blocks. Start by putting 3 ones blocks in front of the student and ask for the value. Do the same with 3 tens blocks and then 3 hundreds blocks. Write the numeral 333 and help the student understand that each 3 has a different value based on its position in the number. Do some modeling of three-digit numbers with the students, pointing out that the place farthest to the right does have a value of 1, but that the value of each place increases as you move to the left. The next place to the left has a value of 10 and the place to the left of 10 has a value of 100. Give students three-digit numbers and have them model them with base-ten blocks and give the value of each digit in the number.

- **Students ignore zeros in a number.**
  
  Students who think that zeros in a number don’t mean anything may think of a number such as 507 as being equal to 57. These students need more practice representing numbers with base-ten blocks. Help them to understand that when there is a 0 in the tens place, it is just as important as any other digit in the tens place because it tells us that there are 0 tens. So for example, the 0 lets us know that 507 is five hundred seven rather than fifty-seven. Have students complete exercises where they represent a number like 507 with base-ten blocks and then represent a number like 57 with base-ten blocks. Ask them to talk about what differences they see.
Around grade 5, they can generalize that the value of a digit is ten times the value of the previous digit as they move to the left.

**PATTERNS IN THE PLACE-VALUE SYSTEM**

The Base-Ten number system was developed by an Indian mathematician in the 7th century. Unlike the number systems that came before it, it was designed to represent every whole number using only 10 unique symbols. This system was also the first system to include a symbol for the value of 0. Place value is the underlying concept in the way we write numbers in the base-ten system. In second grade students begin to see the patterns in this system. They see that they always bundle 10 of something to make 1 in the next place. For three-digit numbers, students need to understand the equivalence of 10 tens and 1 hundred. The equivalence between 10 of a place value and 1 of the next greatest place value is why the number system is called the ‘base-ten’ system.

**ZEROS IN THE TENS PLACE**

It can be difficult for students to understand zeros when they are in the tens place. Students who have difficulty with this often think of zero as having no value, when 0 does in fact have a value. Zero on its own may appear to have no value, but when 0 is one of the digits in a number, it is easy to see its value. For example, $305 is definitely more money than $35. Representing numbers with 0 in the tens place using base-ten blocks can help students understand the role 0 plays when it is in the tens place. Another way to help students understand the value of zero is to have them think, write, and talk about what the world would be like without 0.

**TEACHER TIPS**

Giving students place-value riddles can be very helpful in developing their understanding of place value. Start with easier riddles and then work up to harder ones. Here are some examples:

- I have 4 ones, 14 tens, and 6 hundreds. What number am I?
- I have 2 hundreds, 12 tens, and 25 ones. What number am I?
- I have 40 ones and 5 hundreds. What number am I?
- I have 2 hundreds and 6 ones. What number am I?
- I am between 100 and 200. I have 5 tens and 13 ones. What number am I?

After students have some experience with the riddles you give them, have them write their own.