COMMON MISCONCEPTIONS

- Students focus on the individual digits in the dividend rather than the whole number. Understanding the standard algorithm is difficult for some students because it involves many steps. To make a division problem less complicated, students focus on individual digits. For instance, with a problem such as 583 ÷ 4 they ask ‘how many times does 4 go into 5’ rather than thinking of the individual steps of finding equal shares of 4 in 5 one hundreds, 8 tens, and 3 ones.

- If at any step in the algorithm the divisor is bigger than the dividend, you can move an additional one digit to the right and proceed with division without recording the value that you skipped. For example, a student will erroneously divide 642 ÷ 6 = 17 rather than the correct answer 642 ÷ 6 = 107. Using lines to mark place-value columns can help with this common misconception.

- There is no need to learn the standard algorithm since other strategies such as partial quotients, area models, repeated subtraction, etc. work just as well and are easier to remember. These other division strategies are not as flexible as the standard algorithm especially for problems that involve larger dividends and multi-digit divisors.

DIVISION BY A ONE-DIGIT DIVISOR

Students have explored a variety of strategies for division: area models, repeated subtraction, partial quotients, and the fair share method. The standard algorithm for long division is a tally system arranged in place-value columns to perform the same division operations as in the partial quotients method. Long division starts with the left-hand or largest place value.

Begin by dividing a three-digit dividend by a one-digit divisor. For example, introduce the problem 763 ÷ 5. The repeated series of steps and recording scheme—divide, multiply, subtract, bring down—is not completely intuitive, so students require explicit modeled instruction and practice.
Repeat the modeled instruction with a problem that involves dividing a four-digit whole number by a one-digit whole number. Simultaneously completing a partial quotients solution and the standard algorithm solution can help students make connections between a process they already understand and the standard algorithm way of notation.

Provide examples that place zeros in different positions in the dividend and quotient. For example, students commonly miss zeroes in the middle of a quotient and write 432 ÷ 4 = 18 (instead of 108). Emphasize that the division at the tens place results in a 0 in the quotient in the tens place.

**DIVISION BY A TWO-DIGIT DIVISOR**

Division with one-digit divisors provides the basis for division with two-digit divisors. Next, demonstrate a problem that involves division of a three-digit number by a two-digit number. This problem presents more of a challenge. You can start with a problem such as 779 ÷ 19.

Students will likely have single-digit multipliers memorized but once the divisor surpasses 12, students will struggle to quickly consider multiples. Encourage students to list multiples of the divisor before they start working, to use as they work through the repeated divide and multiply steps. Emphasize going slowly, working the algorithm step-by-step, and being careful with each basic operation within the standard algorithm: division, multiplication, and subtraction.

Students begin the algorithm by moving from the hundreds to the tens place and asking what number times 19 is less than or equal to 77? The multiples list can help them see that 4 × 19 = 76. The subtract and bring down steps result in 779 ÷ 19 = 41.

**MULTI-DIGIT DIVIDENDS AND DIVISORS**

Next, introduce problems involving larger dividends and divisors. Emphasize that although the problems look difficult, if students tackle them step by step while being careful with each of the basic operations, they should have no problem. You may want to begin with a problem that uses work from a prior problem so students can learn the algorithm without too many computations. Present a problem such as 70,072 ÷ 19.

In the prior problem, students listed the multiples of 19, which they can also use for this problem. Walk them through the repeated steps of the standard algorithm as follows.
During class discussions, call on different students to explain the individual steps in the algorithm using the appropriate terminology that connects to the concept.

Use estimation by rounding the dividend and divisor to compatible numbers to check answers. For example, we can estimate the solution to the problem $2,970 \div 11$ with $3,000 \div 10 = 300$. Have students practice estimating to check their answers for reasonability.

Use blank division charts with wide place-value columns to allow students to keep track of regrouping during the steps of the standard algorithm. At a minimum, spread out the digits in the dividend when writing down the problem.