COMMON MISCONCEPTIONS

- The number line always starts at 0.
  Help students to recognize, through varied problems, that the labels on a number line can begin and end at any two numbers. The line can also be partitioned in different ways, to represent the given problem.

- You move left on the number line for addition and right on the number line for subtraction.
  Emphasize the concept that numbers on a number line always increase from left to right and decrease from right to left. Therefore, it makes sense that you move to the right when adding numbers and to the left when subtracting numbers. Encourage students to connect jumps on the number line to counting on and counting back.

- Students focus on the hash marks or numerals on a number line instead of the spaces.
  You can see that a student has this misconception when their answers are consistently off by one. Place emphasis on the unit length. Students need to understand that a number line is like a ruler in that consecutive whole numbers are one unit apart.

COMPARING AND ADDING USING THE NUMBER LINE

Building on their understanding of length, in this lesson students represent whole numbers as lengths on a number line. They can use a notebook or grid paper to make their own number lines. Have students mark and label a line on paper with whole-number units that are equally spaced. Begin by having them compare two numbers. For instance, Jesse and Jorge each built a block of towers. Jesse’s tower is 18 centimeters tall and Jorge’s tower is 11 centimeters tall. Have students use a number line to answer the question “Whose tower is taller?” Help them to see that numbers to the right of a given number on the number line are greater.

Next, have students represent addition on a number line. Model an addition problem such as 49 + 5. Start at 0 and show the jump to 49 to emphasize that 49 represents not just a tick mark, but 49 single-unit lengths from the number 0. From the number 49, have students draw a curve to 50, then continue drawing single-unit curves to 54. Drawing the curves or making the “hops” or “jumps” helps emphasize the concept of length.

Help students to see addition as a process of increasing or putting together. Introduce or review the concept of skip counting, a method of counting forward by numbers other than 1 (for example, by 2s, 5s, 10s, or 100s).

On a football field, Pepe starts at the 10-yard line. He runs 25 yards up the field, pauses, and runs 11 more yards up the field. Which yard line is Pepe on now? [46-yard line] How far did he run? [25 + 11 = 36 yards]
OPEN NUMBER LINES AND SKIP COUNTING

Introduce increasingly complex problems and direct students to use an open number line. The open number line allows students to use their intuition about starting points on the number line and skip counting strategies appropriate to a given problem. The ‘open’ (or ‘empty’) number line also allows for flexibility in extending counting strategies from counting by ones to counting by twos, tens, or hundreds all on the same-sized open number line.

Illustrate how an open number line and skip counting can be used to add 39 + 23.

39 + 23 = 62

Encourage classroom discussions about the different strategies for selecting a starting point and end point for the number line as well as the skip counting strategies.

SUBTRACTING USING THE NUMBER LINE

Next, have students represent subtraction using a number line. Have students begin with a number line with whole-number units that are equally spaced. Present a simple subtraction problem, such as: The red colored pencil is 17 centimeters long. The green colored pencil is 9 centimeters shorter than the red colored pencil. How long is the green colored pencil? Tell students that we can solve this problem by using a number line to subtract 17 – 9. Remind students that when we subtract, we start at the first number and hop left, and so we start at 17 and make 9 single hops to the left, to land on the number 8. Therefore, 17 – 9 = 8.

Move to increasingly complex subtraction problems and direct students to use an open number line. For instance: Eduardo’s family took a two-day drive to his grandparents’ home. They drove 254 miles on Thursday. They drove 125 fewer miles on Friday than on Thursday. How many miles did Eduardo’s family drive on Friday? Have students use an open number line and encourage a discussion of what numbers to write and where to place them on the open number line. For subtraction problems, we put the minuend or a number greater than but close to the minuend near the right end of the open number line, leaving room to “hop” left. Discuss skip counting strategies. Pose the question: What is the fewest number of hops we can make using skip counting? Answer: We can use one 100-mile hop, one 20-mile hop, and one 5-mile hop. Other strategies include using one 100-mile hop, two 10-mile hops, and five 1-mile hops. Allow students to see that no matter the skip counting strategy, we find that 254 – 125 is 129. Eduardo’s family drove 129 miles on Friday.
The number line is an easy model to understand and has great advantages in helping students understand the relative magnitude and position of numbers, as well as to visualize operations. Working effectively with the number line model, students can develop powerful intuitive strategies for single- and multiple-digit addition and subtraction.

One foundational concept of this lesson is that the zero point on a number line is the beginning of the total length and each number on the number line indicates the number of length units each number is from zero.

An important teaching point to convey about the number line is the notion that, unlike a ruler, it is open and flexible. Given this concept, students should recognize that they need to create their own actions on the number lines to give the model meaning. Provide varied problems and ample opportunities for students to construct and partition a number line as they see fit.

Help students to see addition as a process of increasing or putting together and subtraction as taking away or finding the difference. Students should add and subtract first with a labeled number line rather than an open number line.

There many digital tools available for the number line that could accompany this lesson.