COMMON MISCONCEPTIONS

- **A greater denominator represents a greater amount.**
  If students believe that the denominator indicates the size of a whole or how many wholes there are, they may think that a greater denominator represents a greater amount. This indicates a bigger misconception about the meaning of fractions in general and students should review the fraction basics. Using visuals, have students practice identifying that the denominator represents how many equal parts the whole has been split into, and that if the numerator is less than the denominator, we have less than one whole. Given different denominators with accompanying visuals, have students make the connection between piece size and denominator size.

- **You can find a common numerator and select the greater denominator as the greater fraction.**
  While finding common numerators is a valid way of comparing fractions, students likely use this method incorrectly by assuming that a greater denominator indicates a greater fraction. For example, between \(\frac{2}{6}\) and \(\frac{2}{5}\), students say that \(\frac{2}{6}\) is the greater fraction. Remind students that when we compare amounts, we need to compare units of the same size. Since the denominator represents the size of the unit, we want to compare fractions that have the same denominator. Then, the numerators tell us how many of the same-sized units we have. You may also represent \(\frac{2}{6}\) and \(\frac{2}{5}\) visually.
  
  Because fifths are bigger than sixths, they can see that \(\frac{2}{5}\) is the greater fraction. However, emphasize that we ideally want to compare units that have the same size.

- **To find a common denominator for two fractions, only multiply the numerator or denominator by the opposite denominator.**
  Students may not have grasped the reason behind multiplying each fraction by the other fraction’s denominator and may do it incorrectly. To begin, remind students that the reason we multiply the numerator and denominator by the same number is so that we do not change the amount. Give students a visual example, such as showing \(\frac{1}{2}\) and using a line to create 4 total parts.
  
  Indicate to students that the amount has not changed. The numerator has doubled, because we have twice as many pieces, and the denominator has doubled too, because our pieces are half their original size.
  
  To show them why fractions are multiplied by each other’s denominators, you can give them another visual example. Given two identical rectangles, we want to compare \(\frac{1}{3}\) to \(\frac{1}{2}\), but this is difficult while the denominators (or equal-sized pieces of a whole) are different sizes. To make them the same size, we need to cut each of the pieces from one rectangle to match the cuts on the other rectangle (red pen). Now the pieces sizes are the same, and we can compare how many pieces each rectangle has.
COMPARISONS USING BENCHMARK FRACTIONS

Before students learn to compare fractions using common denominators, they first compare two fractions to a third, more familiar fraction, called a benchmark. Usually, the benchmark fraction can be \( \frac{1}{2} \). Given two fractions such as \( \frac{2}{6} \) and \( \frac{3}{4} \), we can show which one is greater without finding a common denominator for \( \frac{2}{6} \) and \( \frac{3}{4} \). Instead, if we can show that \( \frac{1}{2} \) is equal to \( \frac{3}{6} \) and \( \frac{2}{6} \), it becomes clear that \( \frac{2}{6} \) is less than \( \frac{1}{2} \) while \( \frac{3}{4} \) is greater than \( \frac{1}{2} \). Then we know that \( \frac{3}{4} \) must be greater than \( \frac{2}{6} \). Other benchmark fractions can be used, such as using \( \frac{1}{4} \) to compare \( \frac{1}{8} \) and \( \frac{1}{2} \), but at this point it is more efficient for students to move on to learning to find common denominators.

COMPARING FRACTIONS WHOSE DENOMINATORS ARE MULTIPLES

The first time that students are introduced to comparing fractions with different denominators, present them with two fractions, only one of which has to be converted in order to find common denominators. For example, given \( \frac{5}{6} \) and \( \frac{2}{3} \), we only need to convert \( \frac{2}{3} \) to \( \frac{4}{6} \), and then we can easily compare \( \frac{5}{6} \) and \( \frac{4}{6} \). For the first examples using this method, provide two fractions that are either both greater than \( \frac{1}{2} \) or both less than \( \frac{1}{2} \). This way, they are not able to use the benchmark method and they discover the need for another way to compare.

FINDING A COMMON DENOMINATOR

Next, present students with two fractions with denominators that are not multiples of one another. We need to find a third common denominator to compare the fractions. Begin the process by listing equivalent fractions of each fraction, until we reach a common denominator. Once students have had some practice using this method, show them that we can skip the tedious process of listing several equivalent fractions, and instead to multiply the numerator and denominator in each fraction by the denominator of the other fraction. For example, to find a common denominator for \( \frac{2}{6} \) and \( \frac{1}{4} \), students can simply multiply \( \frac{2}{6} \times \frac{4}{1} = \frac{8}{24} \) and \( \frac{1}{4} \times \frac{6}{6} = \frac{6}{24} \). Using this method, they do not always find the lowest common denominator, but this is not a concept they have learned yet. If students ask about other equivalent fractions like \( \frac{3}{12} \) and \( \frac{3}{12} \), show them that those equivalent fractions are also valid. The denominator of 24 is simply easier to find at this stage.

TEACHER TIPS

Ensure that students see not only examples that use round objects, like pies and pizzas, but also that they see rectangular objects for their models.

Give students opportunities to make decisions regarding which method they should use to compare fractions. When is it simplest to use benchmark fractions? When do benchmark fractions not help us? Do we need to change both fractions to find the common denominator, or do we only need to change one? Should we list all the fractions, or should we multiply by denominators? Allow students to explore the usefulness and efficiency of each method as they learn.