COMMON MISCONCEPTIONS

- Students fail to understand the place value of the digits when adding or subtracting whole numbers.

Students who lack place value understanding will add and subtract without regard to place value. One indication that a student lacks place value understanding is when the student does not properly stack the two numbers—that is, the student does not vertically align the numbers by place value. This may be more likely to happen when adding a 2-digit number to a 3-digit number. Explain to students that we line up the right-most place value position (the ones) in our two numbers when we stack them to add or subtract.

- Students subtract the smaller digit in a column from the larger, regardless of which appears in the top number.

Students need to recognize that when subtracting whole numbers, we subtract the second number from the first number and, using the standard algorithm, we stack the first number on top. Working from right to left, we subtract the bottom digit from the top digit in each place value column. If in any column, the top digit is smaller than the bottom, this is an indication that we must regroup in order to subtract.

- Students record the regrouping process in the written form without understanding what the process represents conceptually.

The most important concepts underpinning the standard algorithm are place value and regrouping. It is important that students understand why we exchange 10 in one place value position for 1 in the position to the left or the reverse, exchanging 1 in one place value position for 10 in the position to the right. We use regrouping (composing and decomposing base ten units) to help us carry out our computations—that is, we regroup in addition when the sum in a column is greater than 9 and in subtraction when the minuend is smaller than the subtrahend.

ADDING USING THE STANDARD ALGORITHM

There are a series of steps that students should following when completing the standard algorithm for addition:

1. Stack the numbers vertically by matching the place values.
2. Add together the numbers that share the same place value, beginning with the ones place.
3. Write the sum below each column.
4. If the sum of any one column is greater than nine, regroup by exchanging 10 in that column (place value position) for 1 in the column (place value position) to the left.
It is important that students are presented with addition problems in increasing difficulty, starting with no regrouping required and ending in regrouping being required for all place values. This will allow students to learn the process without being overwhelmed by additional regrouping steps early in the process.

**SUBTRACTING USING THE STANDARD ALGORITHM**

The general approach to developing the subtraction algorithm is the same as for addition. But instead of exchanging 10 in one place value position for 1 in the position to the left, you will exchange 1 in one place value position for 10 in the position to the right.

The steps to the standard algorithm for subtraction are as follows:

1. Stack the numbers vertically by matching the place values. Be sure to put the first number on top.
2. Working from right to left, subtract the bottom number from the top number.
3. Put the answer at the bottom of each column.
4. As you work from right to left, if the bottom number in a place value column is larger than the top number, you need to regroup by exchanging 1 from the place value position to the left for 10 in the place value column you are working in.

Like addition, present students with subtraction problems that first require no regrouping to subtract. Then introduce problems where regrouping is required in 1 then 2 place values.

One type of multiple place value regrouping is one where the place value the student tries to regroup from is 0, such as: 3,403 – 1,138 = __________. Tell students that if they encounter a zero when regrouping, just continue to the left until you find a non-zero number.

Some students will want to subtract the smaller digit in a column from the larger, regardless of which appears in the top number. Tell students that as they work through the steps for each column to ask themselves: is the number on top larger than or equal to the number on the bottom? If the answer is no, that should be a signal that they need to regroup.

**TEACHER TIPS**

Students have been building an understanding of addition and subtraction since kindergarten and first grade. In third grade, they fluently add and subtract within 1,000 using methods based on place value, properties of operations, and/or the relationship between addition and subtraction. In 4th grade, students will be expected to be proficient using the standard algorithm since it is more efficient than other strategies for adding and subtracting large whole numbers and, later, decimals.

The primary focus in teaching the standard algorithm is not only having students memorize a series of steps, but having students understand that it is a shorthand form of writing a process that they already know and understand (adding by place value). Although students in 3rd and 4th grades will be generally moving away from strategies for addition and subtraction that rely on concrete models, a review of these models before the lesson may help you make explicit the connection between the concept of regrouping and the steps of the standard algorithm procedure.

When teaching the standard algorithm, provide students with plenty of time to work through increasingly challenging problems. Encourage students to explain what they did and why as they work through the steps. One or two problems in a lesson with lots of discussion will be more productive than lots of problems applying rules that students do not conceptually grasp.

Provide students with opportunities to review problems where one or more of the steps using the standard algorithm are incorrect. The use of error analysis will be beneficial in the learning process.