COMMON MISCONCEPTIONS

• The larger number is always divided by the smaller number.
A common misconception among students is that when dividing you always put the larger number first. That is, they believe that a smaller number cannot be divided by a larger number. Working on equal sharing problems helps students to see that in certain situations, the equal share is less than one whole. This is the result of the dividend being smaller than the divisor. It is important to emphasize that the number of items to being shared can be less than the number of sharers.

• Students do not understand the meaning of the numerator and denominator in real-world situations.
When interpreting fractions as division, students do not make the correct distinction between the numerator and denominator. That is, that the numerator represents the number of items or amount being shared while the denominator represents the number of sharers. This is especially true when students solve word problems where the first number mentioned is the denominator value (for example, ‘write the fraction that represents 5 students sharing 3 cookies’). Students may put the numbers in the order they were presented and represent this situation with the expressions $5 \div 3$ and $\frac{5}{3}$. Again, working with equal sharing activities and emphasizing the role of numerator and denominator in the fraction that represents the situation is critical.

• Students struggle to convert between fractions and mixed numbers.
Some students struggle with problems that involve answers that are mixed numbers. For example, students may recognize that if 8 friends share 27 cookies, each friend receives 3 full cookies by partitioning 24 cookies into 8 groups of 3 cookies. But they do not understand how to partition or equally subdivide the 3 leftover cookies among the 8 friends. Visual models that partition either all 27 cookies or the 3 leftover cookies into 8ths can help students to understand this subdivision of the wholes.

INTERPRETING FRACTIONS AS DIVISION

The concept of fractions as division can be introduced to students by building upon their knowledge of division—established in early grades—as the concept of equal sharing. Students have prior knowledge of simple sharing task such as: Two friends share four candy bars equally. How many candy bars does each friend get? This problem can be solved with the equation $4 \div 2 = 2$. Each friend gets 2 candy bars.

Now we introduce the concept of four friends sharing one pizza. Introduce the concept using concrete and visual models. This problem can be modeled using fraction circles, which are particularly suited to the context of sub-diving pizza.
Explain that we have divided one pizza into four equal-sized pieces. With a visual, we can now connect this to the division expression $1 \div 4 = \frac{1}{4}$. Next, we can explore dividing two pizzas equally among four friends. Each friend receives $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ of a pizza.

Building from the visual to the equation is very beneficial to students’ understanding. Our first example shows that $1 \div 4 = \frac{1}{4}$ and our second example shows that $2 \div 4 = \frac{2}{4}$. The division expression can be written as a fraction with the same digits. The fraction looks a lot like the division expression and represents the amount that each person receives. Explain that we can write division expression as fractions. The numerators are the whole amounts that are being shared or divided and the denominators are the number of equal parts or how many are sharing the whole.

**INTERPRETING FRACTIONS AS DIVISION**

As you continue to build upon the concept of fractions as division, introduce more challenging problems. Problem difficulty is determined by the relationship between the number of things to be shared and the number of sharers. So for example, have students consider how 4 friends can share a 3-foot-long submarine sandwich. Explore this problem with a different representation or model, such as a fraction strip, which—like fraction circles discussed in Topic 1—allows the model to match the concept (i.e. submarine sandwich and long strip).

Using a fraction strip allows us to partition each one-foot segment of the sandwich into 4ths. Each person gets $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ of a foot of the sandwich. Once you have presented the problem visually you can relate the division expression $3 \div 4$ and the fraction $\frac{3}{4}$ to the model.

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**INTERPRETING FRACTIONS AS DIVISION – USING THE ALGORITHM**

Next, introduce problems where it is practical to use equivalent fractions. For example, if a class of 40 students shares 5 pizzas, what is each student’s equal share of the pizzas? Students may recognize that they can use the division expression $5 \div 40 = \frac{5}{40}$ and conclude that each student would get $\frac{5}{40}$ of a pizza. Discuss with your students the practicality of serving the pizzas in fortieths. Ask, “How can I make 40 equal parts out of 5 pizzas so that each student gets $\frac{5}{40}$ of a pizza? Is there another way to represent the value $\frac{5}{40}$ so that we don’t cut each pizza into 40 slices?” This question leads to thinking about equivalent fractions. The simplified answer of $\frac{1}{8}$ of a pizza per student makes more sense.

Have students explore problems where equal portions are greater than one whole, so the leftovers need to be subdivided. For example, if we want to load 23 tons of dirt equally into 3 pick-up trucks, one solution is to partition 7 tons into each truck ($7 \times 3 = 21$), leaving 2 tons to split equally among the 3 pick-up trucks. Each truck gets 7 tons plus an additional $\frac{2}{3}$ ton of dirt. Another solution is to partition each of the 23 tons into thirds, so each truck receives $23 \times \frac{1}{3} = \frac{23}{3}$ tons of dirt. Relate each scenario to the division expression and fraction or mixed number answer: $23 \div 3 = \frac{23}{3}$, or $3 \frac{2}{3}$. 
Fractions have many different meanings (part-whole, ratio, division), and in the early elementary grades it is the part-whole meaning that is emphasized. Understanding fractions means understanding all the possible concepts that fractions can represent, and it is important that students understand how and why fractions represent division.

Have students explore fraction as division with a variety of models—area, set, length— and connect the models to the symbolic representation. Repeating the same sharing activity with a different representation may be helpful to students’ understanding.

Instruction with fractions often relies on rote memorization of rules and algorithms. It is best to build up to the algorithm using concrete and visual models as stepping-stones.