COMMON MISCONCEPTIONS

• The order operations are applied does not matter.
  Students often think that you can do the operations in any order in an expression, and no matter what the order, you will always get the same answer. The activities and problems in this lesson are designed to help students see that this is not true. Highlight to students examples where the result is different when operations are applied in different orders, such as $6 + 6 ÷ 3$.
    • Division first: $6 + 6 ÷ 3 = 6 + 2 = 8$
    • Addition first: $6 + 6 ÷ 3 = 12 ÷ 3 = 4$
  Also reinforce with students why it is important that there is a rule for the order operations are applied, giving examples of important applications of mathematics.

• When evaluating an expression, do multiplication before division and then addition before subtraction.
  When students use PEMDAS (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction) to remember the order of operations they may think multiplication must be done before division and addition must be done before subtraction. Many teachers avoid using PEMDAS for this reason. Reinforce with students that for multiplication/division and addition/subtraction the operations must be done in order from left to right.

• You don’t need to use order of operations within parentheses.
  When expressions within parentheses are introduced, students often either (a) do not apply order of operations within the parentheses or (b) apply order of operations as if the parentheses are not there. To help students understand how to evaluate expressions with parentheses, encourage students to write the expression within the parentheses by itself and evaluate it on its own. Then, have them copy the evaluation back into the original expression.

SEQUENCE OF LEARNING ORDER OF OPERATIONS

In grade 3, students use order of operations. They work with expressions with two operations and without grouping symbols. They use the order of operations rule of moving left to right and doing multiplication and division before addition and subtraction. In grade 5, students use parentheses and brackets to indicate order and work with expressions with more than two operations.
DIFERENCE BETWEEN AN EQUATION AND AN EXPRESSION

Students sometimes get confused about the difference between an expression and an equation. An expression is a mathematical sentence with numbers, variables, and/or operations. Students are primarily working with expressions in this lesson. An equation has an equal sign separating two expressions or showing the value of an expression. 

\[(3 + 2) \times 4\] is an expression. \[3 + 2 \times 4 = 5 + 1\] is an equation.

GROUPING

When solving real-world problems, multiple levels of grouping may be required to force operations to be applied in the correct order to solve the problem. Generally, parentheses are used as the first level of grouping and brackets used as the second level of grouping. For example, \[38 - [2 + 5 \times (3 + 4)]\] states that first 3 and 4 must be added (first level grouping) then, the sum multiplied by 5 and added to 2 (second level grouping) before being subtracted from 38.

In more complex expressions grouping may be implied. For example, when an expression is written as a fraction in vertical format, the terms in the numerator are grouped together and the terms in the denominator are grouped together. For example, \[\frac{3 + 5}{8 \times 2}\] = \[(3 + 5) ÷ (8 \times 2)\]

TEACHING TIPS

It is a good practice to switch where you place the multiplication and division (and addition and subtraction) symbols. This helps students get into the habit of looking for these symbols from left to right. Many teachers do not use PEMDAS (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction) as a device for remembering the order of operations because it tends to make students think that multiplication comes before division and addition comes before subtraction.